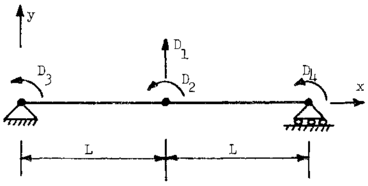

Fig. 1 Simply supported beam.

eliminating the two rotational degrees of freedom at the ends of the beam. Using the consistent mass and stiffness matrices for the beam elements[2] and the reduction method[1,3] to eliminate degrees of freedom D_3 and D_4, yields the eigenvalues

$$\omega_1{}^2 = (105/17)EI/\rho AL^4 \quad \omega_2{}^2 = 157.5\, EI/\rho AL^4 \tag{4}$$

The auxiliary eigenvalue problem, Eq. (2), which establishes convergence of Eq. (1) yields the double root solution

$$\bar{\omega}_1{}^2 = \bar{\omega}_2{}^2 = 420\, EI/\rho AL^4 \tag{5}$$

Clearly, the frequencies of Eq. (5) are all greater than the frequencies of Eq. (4); therefore, the series expansion of Eq. (1) will converge for each frequency of Eq. (4) and the reduction method has been properly used.

To recover the eliminated degrees of freedom, the back-transformation approximation, Eq. (3), is used and the resulting modal vectors are shown in Table 1 along with the modal vectors obtained from the Guyan method[3] and the exact method of Ref. 4. As can be seen from Table 1, the back-transformation of Eq. (3) produces excellent results: an error of -0.013% for the first mode and an error of $+7.42\%$ for the second mode. Without including the inertia terms in Eq. (3), which is the Guyan back-transformation, the errors are -4.5% and -50% for the first and second modes, respectively.

Table 1 Modal vectors for example problem of Fig. 1

Degree of freedom	Present analysis first mode	second mode	Guyan method first mode	second mode	Exact (Ref. 4) first mode	second mode
1	1	0	1	0	1	0
2	0	1	0	1	0	1
3	$1.5706/L$	-1.0742	$1.50/L$	-0.50	$1.5708/L$	-1
4	$-1.5706/L$	-1.0742	$-1.50/L$	-0.50	$-1.5708/L$	-1

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Comment on "On Multiple-Shaker Resonance Testing"

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THE characteristic phase-lag method attributed by the author[1] to Bishop and Gladwell was first published in 1948[2] and influenced most of Traill-Nash's early work. Its application to multiple-shaker resonance testing was the subject of an AGARD report[3] in which a principle of minimum reactive energy and the essentially equivalent complex admittance rule is demonstrated. The main result, however, is that, for a limited number of shakers and small enough damping, the forcing amplitude ratios should be such that phase resonance occurs at each shaking point.

The GRAMPA (and later MAMMA) multiple-shaker installation, described in the author's Ref. 6, was based on this result. The performance obtained at Royal Aircraft Establishment with this apparatus was so convincing that it would be interesting to compare this method of regulating the shaker amplitudes with the various approaches described in the article. It may also be mentioned that the characteristic phase-lag and damping theories, duly quoted in many books and articles on the subject of forced response of linear damped systems, were later enlarged and the subject of further publications.[4-6]

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[4] Fraeijs de Veubeke, B., "Aircraft resonance testing," Chap. 3, Pt. I, AGARD Manual of Aeroelasticity.

[5] Fraeijs de Veubeke, B., "Les déphasages caractéristiques en présence de modes rigides et de modes non amortis," *Académie Royale de Belgique, Bulletin de la Classe des Sciences*, 5e Série-Tome LI, 1965, pp. 525–540.

[6] Fraeijs de Veubeke, B., "Analyse de la réponse forcée des systèmes amortis par la méthode des déphasages caractéristiques," *Revue Française de Mécanique*, Vol. 13, 1965, pp. 49–58.

Reply by Authors to B. Fraeijs de Veubeke

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THE authors express their appreciation to Fraeijs de Veubeke for supplying references to his own work on resonance testing. It was not the authors' intention to present a complete history of the development of resonance test methods. The Bishop and Gladwell reference[1] was quoted because it provides this background as well as an extensive list of references, including references to Fraeijs de Veubeke's 1948[2] and 1956[3] works on the "characteristic phase lag theory."

The technique for determining excitation frequency and force amplitudes in a multiple-shaker test, referred to (perhaps erroneously) by the authors as the "Asher method," had been discussed by Fraeijs de Veubeke, Asher,[4] and others. However, the authors sought, through use of a simulation study, to emphasize the fact that an insufficient number of shakers or an inappropriate group of shakers could produce spurious natural frequencies and unacceptable mode shapes, and to explore more

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